

EXTENSION 1

Stage 6



Calculus (Ext1), C3 Applications of Calculus (Y12), Differential Equations (Ext1)
Equations and Slope Fields

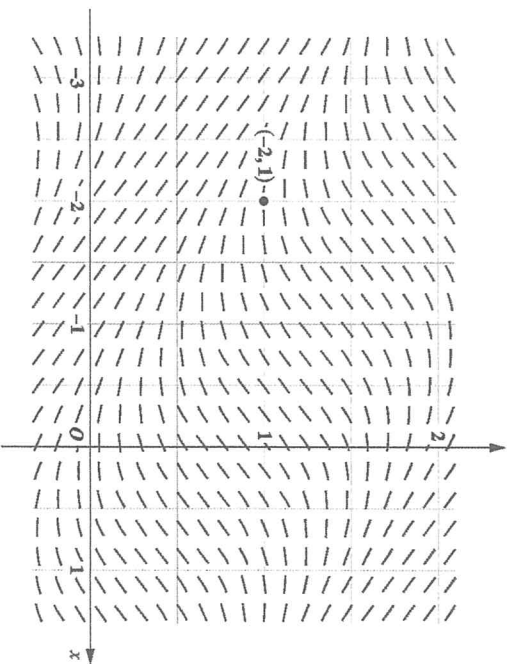
Teacher: Stewart Mitchell

Exam Equivalent Time: 24 minutes (based on allocation of 1.5 minutes per mark)

Questions

1. Calculus, EXT1 C3 2023 HSC 3 MC

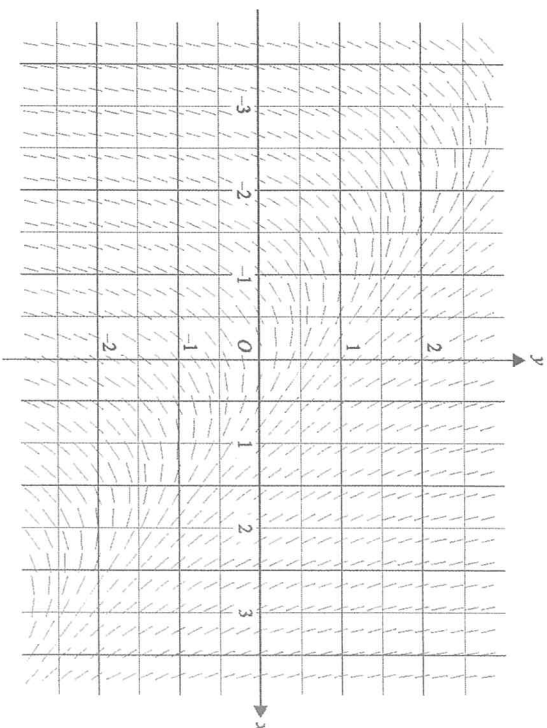
The diagram shows the direction field of a differential equation. A particular solution to the differential equation passes through $(-2, 1)$.



Where does the solution that passes through $(-2, 1)$ cross the y -axis?

- A. $y = 1.12$
- B. $y = 1.34$
- C. $y = 1.56$
- D. $y = 1.78$

2. Calculus, EXT1 C3 2016 SPEC2 10 MC

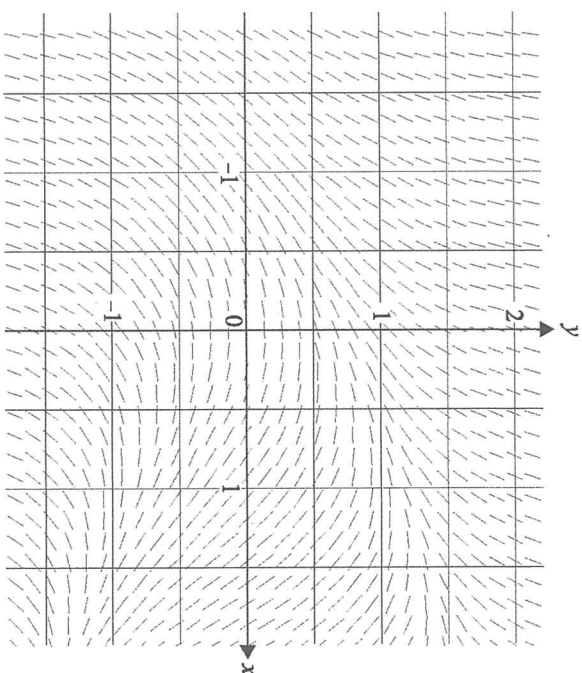


The direction field for the differential equation $\frac{dy}{dx} + x + y = 0$ is shown above.

A solution to this differential equation that includes $(0, -1)$ could also include

- A. $(3, -1)$
- B. $(3.5, -2.5)$
- C. $(-1.5, -2)$
- D. $(2.5, -1)$

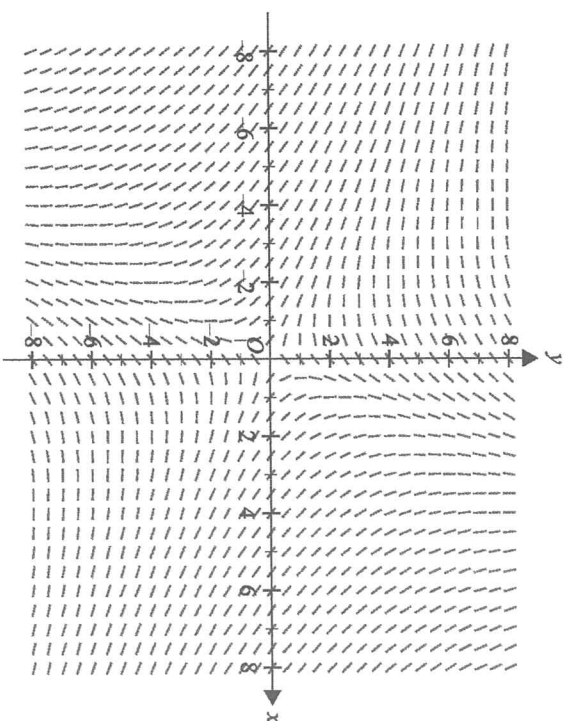
3. Calculus, EXT1 C3 2017 SPEC2 8 MC



The differential equation that best represents the direction field above is

- A. $\frac{dy}{dx} = x - y^2$
- B. $\frac{dy}{dx} = y - x$
- C. $\frac{dy}{dx} = y^2 - x^2$
- D. $\frac{dy}{dx} = y^2 - x$

4. Calculus, EXT1 C3 2018 SPEC2 10 MC

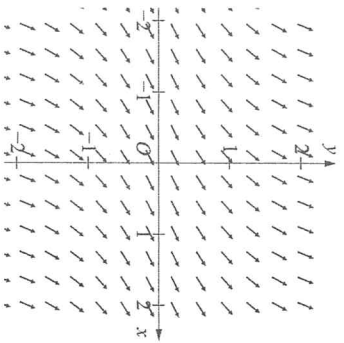
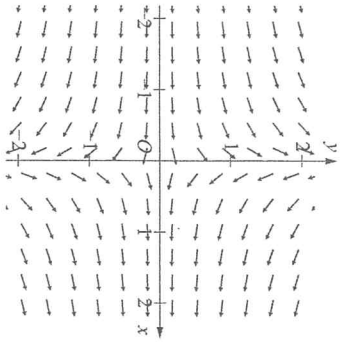
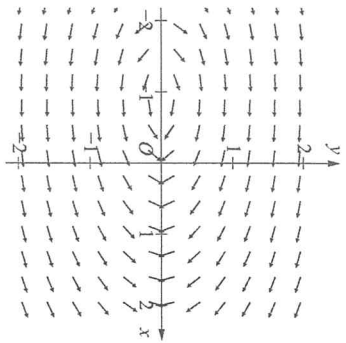
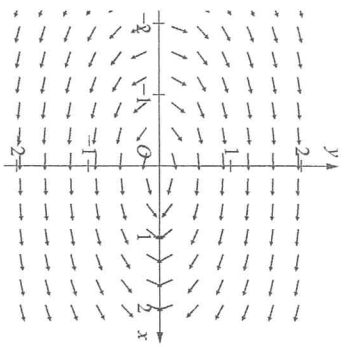


The differential equation that best represents the direction field above is

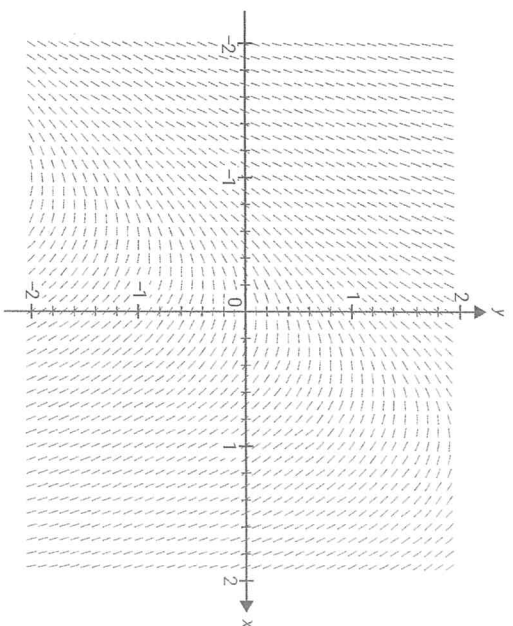
- A. $\frac{dy}{dx} = \frac{2x + y}{y - 2x}$
- B. $\frac{dy}{dx} = \frac{x + 2y}{2x - y}$
- C. $\frac{dy}{dx} = \frac{2x - y}{x + 2y}$
- D. $\frac{dy}{dx} = \frac{x - 2y}{y - 2x}$

5. Calculus, EXT1 C3 2020 HSC 7 MC

Which of the following best represents the direction field for the differential equation $\frac{dy}{dx} = -\frac{x}{4y}$?

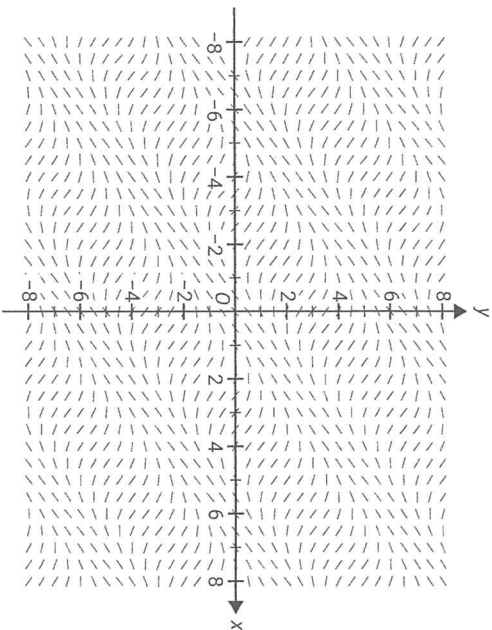


6. Calculus, EXT1 C3 2021 SPEC2 10 MC



The differential equation that has the diagram above as its direction field is

- A. $\frac{dy}{dx} = y + 2x$
- B. $\frac{dy}{dx} = 2x - y$
- C. $\frac{dy}{dx} = x + 2y$
- D. $\frac{dy}{dx} = y - 2x$

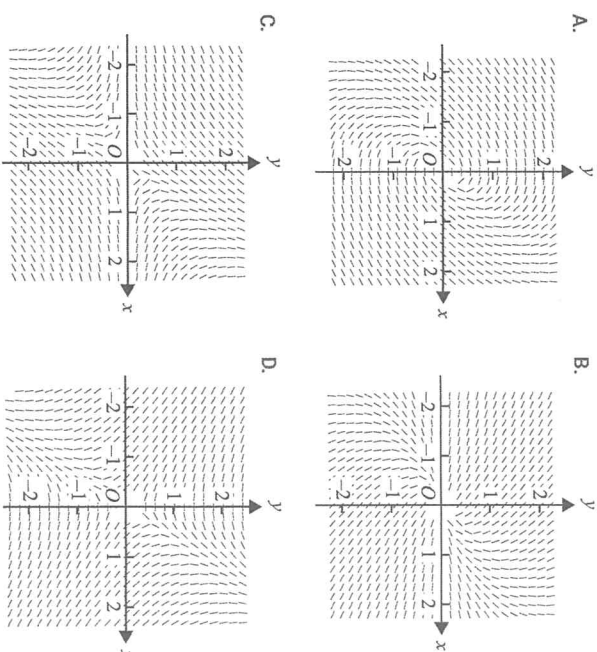


The differential equation that has the diagram above as its direction field is

- A. $\frac{dy}{dx} = \sin(y - x)$
- B. $\frac{dy}{dx} = \cos(y - x)$
- C. $\frac{dy}{dx} = \frac{1}{\cos(y - x)}$
- D. $\frac{dy}{dx} = \frac{1}{\sin(y - x)}$

$P(x, y)$ is a point on a curve. The x -intercept of a tangent to point $P(x, y)$ is equal to the y -value at P .

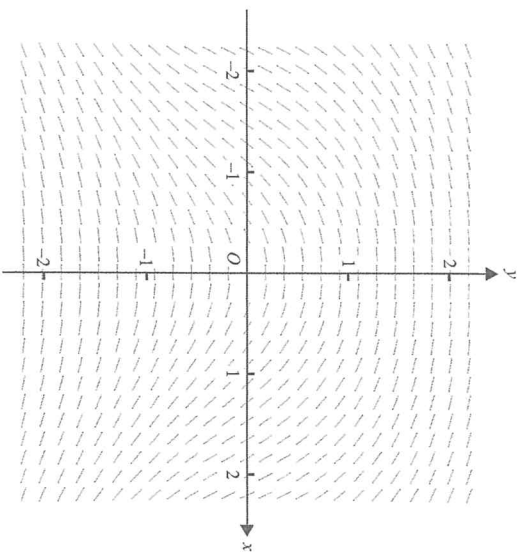
Which one of the following slope fields best represents this curve?



Let $\frac{dy}{dx} = (4 - y)^2$.

Express y in terms of x , where $y(0) = 3$. (3 marks)

A slope field representing the differential equation $\frac{dy}{dx} = -\frac{x}{1+y^2}$ is shown below.



a. Sketch the solution curve of the differential equation corresponding to the condition $y(-1) = 1$ on the slope field above and, hence, estimate the positive value of x when $y = 0$. Give your answer correct to one decimal place. (2 marks)

b. Solve the differential equation $\frac{dy}{dx} = \frac{-x}{1+y^2}$ with the condition $y(-1) = 1$. Express your answer in the form $axy^3 + by + cx^2 + d = 0$, where a, b, c and d are integers. (2 marks)

The direction field for a differential equation is shown below.
 The graph of a particular solution to the differential equation passes through the point P .
 On the graph, sketch the graph of this particular solution. (1 mark)

